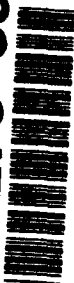


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## OPTIMIZING FLIGHT OPERATIONS FOR AN AIRCRAFT CARRIER IN TRANSIT

Richard E. Rosenthal  
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# **OPTIMIZING FLIGHT OPERATIONS FOR AN AIRCRAFT CARRIER IN TRANSIT**

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*Abstract: Suppose an aircraft carrier is in transit to an assigned position within strike range of a designated target, and is required to be there at a specified time. The carrier may use aircraft assets for defense against threats that may be encountered en route, but doing so will encumber the carrier's progress toward the required objective. We present a highly detailed integer programming model for scheduling aircraft launches and recoveries, so as to achieve an optimal balance between the conflicting needs of self protection and on-time arrival.*

## 1. Background

One of the most mobile and powerful forces available for protecting United States interests is the Aircraft Carrier Battle Group. The battle group, which has the arcane acronym CVBG, has an aircraft carrier at its center and several warships and supply ships surrounding it in formation. The CVBG's greatest advantage is having its own air force available 100% of the time. When combined with a Marine amphibious assault group, a CVBG is capable of handling a variety of air, sea and land combat operations.

We consider the situation when a CVBG is required to move to an assigned position within strike range of a designated target, and must get there within a specified time window. The time restrictions are very important. If the battle group arrives too early, it may forewarn the adversary of impending action. If it arrives too late, it may cause failure of the assigned mission.

Sometimes the CVBG is pre-positioned in the vicinity of the assigned action, so that it can easily get to the right place at the right time. The case of interest in this paper is when the carrier is not suitably pre-positioned and has to make a long transit.

An important concern of the CVBG while in transit is the need to protect itself from potential threats under, on, or above the ocean's surface. The aircraft carrier is usually too far from shore to rely on land-based assets for protection, so it must launch and recover several aircraft while in transit. These flights serve defensive missions such as combat air patrol, early warning and "sanitation/delousing" (searching for threats along the CVBG's intended course). Additional flights are needed for proficiency training and maintenance checks, so that aircrews and aircraft remain at high levels of readiness.

Generally speaking, the CVBG commander would like to maximize the number of flights launched so as to maximize self-protection. However, this objective is in conflict with the need to arrive at the assigned location during the specified time window. The conflict arises because en route flight operations usually require deviations from the carrier's intended course, which is called the *position of intended motion* or *PIM*.

The extent to which flight operations cause deviation from the PIM depends on several factors, including weather conditions (wind direction, wind speed, visibility, cloud ceiling, sea state), time of day (light or darkness) and the number of aircraft needed. The most important factor is wind. If the carrier is headed directly into the wind, then the only change required may be to lower its speed. If, as is more likely, the wind is coming from some other direction, the carrier must alter its course to launch and recover aircraft, thereby increasing the actual distance that must be traveled in order to arrive at the assigned location.

The amount of time the carrier remains off the intended course during flight operations depends on the number of planes involved, the environmental conditions and the time of day. Rough seas, low visibility, and darkness dictate longer time intervals between successive launches and/or recoveries.

Figure 1 illustrates a PIM (a straight line in this case) and an associated track taken over three flight operations cycles. Notice that the final position of the carrier is  $E_3$ , not the assigned position  $F_3$ , thereby jeopardizing the overall mission. This figure illustrates the essential conflict addressed by this paper.

We developed an optimization model embedded in a decision support system for addressing this conflict. The Navy currently schedules flight operations on aircraft carriers manually. As a result, the effects of flight operations on deviations from PIM are sometimes misjudged, causing flight cancellations and consequent decreases in protection and readiness.

## 2. Decision Support System

The decision support system consists of a control program called the *Carrier Optimization Launch Program* and an embedded optimization model called the *Carrier Optimization Launch Model*. The control program provides: a user interface for data input, evaluation capability for manual flight operations plans (fixed decision variables), calculation of approximations for model parameters that must be forecasted, complete interfacing with the optimization model, and output based on standard Navy report formats.

Another feature of the decision support system is to help the flight operations officer deal with the problem of infeasibility. In some cases, the input data calls for a minimum number of flights and an arrival time window that, taken together, are mathematically impossible to achieve. The system not only detects this condition, but also makes recommendations on how to correct the infeasibility. Some possible corrections are: speed and course modifications, reduction in defensive posture, shorter duration flights, and reduction in acceptable transit distance.

The system runs on 386/486-based personal computers. The control program was implemented in Basic and the optimization model in GAMS (the General Algebraic Modeling System, described by Brooke, Kendrick and Meeraus (1992)).

The highly detailed mixed integer optimization model at the heart of the decision support system is the focus of the remainder of the paper.

## 3. Optimization Model

The purpose of the optimization model is to determine a schedule of flight launches and recoveries, so as to maximize defensive posture, subject to arrival at the assigned location within the prescribed time window and other constraints based on physical limitations, environmental conditions, and military judgement.

A key concept in the model is the *flight operations cycle*, or *cycle*, as we will refer to it henceforth. On an aircraft carrier in transit, flights work in cycles, in contrast to an airport, where flights take off and land at any time. Planes are launched and recovered in clusters. The planes that are launched together may not necessarily be recovered together, because they can stay airborne for different numbers of cycles. Usually, a launch cluster is initiated just prior to a recovery cluster. The primary reason for grouping launches and recoveries in tightly interlocking cycles is to minimize the time required for the carrier to steer away from its intended course during flight operations. The secondary reason is to enhance flight deck performance and organization.

One of the key aspects of the model is its consideration of the possibility of sending out aircraft on double- or triple-cycle sorties. Double- and triple-cycle sorties reduce the launch and recovery time and thereby reduce the carrier's deviation from PIM.

### 3.1 Indices

We use the following indices to formulate the optimization model:

$i$  indexes cycles  
 $j$  indexes aircraft types  
 $k$  indexes sortie lengths

Typical values over which these indices range are as follows:

$i \in \{1, 2, 3, \dots, I\}$   
 $j \in \{F-14, F/A-18, A-6, EA-6, E-2, S-3, ES-3\}$   
 $k \in \{SC, DC, TC\}$

where  $I$  is a predetermined integer, and  $SC, DC, TC$  mean *single-cycle*, *double-cycle*, and *triple-cycle sorties*, respectively. The length of each cycle is an input parameter, usually within the range 0.75 to 2.75 hours, and it can vary by cycle.

### 3.2 Decision Variables

The primary decision variables of the optimization model are general integer variables, which, taken together, represent the complete flight operations schedule:

$x_{ijk}$  = the number of aircraft of type  $j$  to launch at the start of cycle  $i$  for sorties of length  $k$ .

In some cases, the flight operations scheduler has previously decided on specific values for some of these launch variables. We refer to these as *non-discretionary* or *fixed* launches, and we refer to launches that are left under control of the optimizer as *discretionary* launches. Data concerning the non-discretionary launches are communicated through the control program and they are treated as fixed variables in the model. Similarly, if some aircraft types are



precluded from double- or triple-cycle sorties, the program eliminates the corresponding variables.

The discretionary launch variables and input data parameters are used to define the objective function and constraints of the model. Several of the model's constraints are *elastic*, meaning they can be violated at a cost given by an input penalty parameter. As a result, the model has additional sets of variables, called *elastic variables*, that represent the amounts of constraint violation. In the constraint formulations to follow, we do not show the elastic variables explicitly, but rather indicate their existence by the use of a small circle over the relational operator ( $\leq$  or  $\geq$ ) of the constraint. Likewise, the penalty terms are subtracted from the objective function but are not explicitly detailed in the displayed formulation.

### 3.3 Objective Function

The objective of maximizing the carrier battle group's defensive posture is represented by maximizing a weighted sum of the number of launches (less the elastic penalties for constraint violation):

$$\text{Maximize } \left( \sum_{ijk} w_{ijk} x_{ijk} - \text{elastic penalties} \right)$$

The weights and penalties are obtained from the commander's and flight operations officer's preferences. Some care must be taken with the weights to prevent one aircraft type from monopolizing all the discretionary flights.

### 3.4 Constraints

There are three fundamental categories of constraints: asset utilization restrictions, defensive posture requirements, and transit requirements. The input parameters used in the formulation of these constraints are introduced as they appear. Most parameters have self-explanatory names. The first three sets of constraints are asset utilization restrictions with elastic violations allowed.

**Limit the number of aircraft of each type that can be launched in each cycle.** The Aircraft Squadron Commander may authorize a maximum number of launches per period of a particular aircraft type for a variety of manpower or equipment based reasons.

$$\sum_k x_{ijk} \leq \text{Max\_Sorties}_{ij}, \quad \forall ij \quad (1)$$

**Limit the total number of aircraft of all types that can be launched in each cycle.** These restrictions are imposed due to the flight deck's capacity, particularly in night cycles, rough seas or low visibility. They may also be motivated by the need to retain some aircraft in reserve.

$$\sum_{jk} x_{ijk} \leq \text{Max\_Launch\_per\_Cycle}_i, \quad \forall i \quad (2)$$

**Limit the total number of flight operations in each cycle.** These restrictions extend the previous set of constraints to include recoveries as well as launches, if the commander so desires.

$$\sum_{jk} x_{ijk} + \sum_j (x_{i-1,j,SC} + x_{i-2,j,DC} + x_{i-3,j,TC}) \leq \text{Max\_Ops\_per\_Cycle}_i, \quad \forall i \quad (3)$$

The fourth and fifth sets of constraints are defensive posture requirements. These constraints are also elastic, so that the model can furnish useful guidance even under infeasible conditions.

**Each type of aircraft must fly a minimum number of flight hours.** These constraints may also be motivated by squadron hour and training requirements.

$$\sum_i [LC_i x_{ij,SC} + (LC_i + LC_{i-1}) x_{i-1,j,DC} + (LC_i + LC_{i-1} + LC_{i-2}) x_{i-2,j,TC}] \geq \text{Min\_Flight\_Hours}_j, \quad \forall j \quad (4)$$

where  $LC_i$  = the length in hours of cycle  $i$ .

**Maintain the required number of airborne planes in each cycle.**

$$\sum_k x_{ijk} + \sum_{k \neq SC} x_{i-1,jk} + x_{i-2,j,TC} \geq \text{Min\_Airborne}_{ij}, \quad \forall ij \quad (5)$$

The sixth and seventh sets of constraints are asset restrictions on airplanes and time, respectively. They are hard constraints, meaning violations are not allowed at any cost.

**Do not exceed aircraft availability.** During cycle  $i$ , the total number of aircraft of type  $j$  scheduled to be launched, recovered, or kept airborne must be within the number available.

$$\sum_k (x_{ijk} + x_{i-1,jk}) + \sum_{k \neq SC} x_{i-2,jk} + x_{i-3,j,TC} \leq \text{AC\_Avail}_{ij}, \quad \forall ij \quad (6)$$

**Do not exceed the time available for launch and recovery operations within each cycle.**

$$LT_i \sum_{jk} x_{ijk} + RT_i \sum_j (x_{i-1,j,SC} + x_{i-2,j,DC} + x_{i-3,j,TC}) \leq \text{Time\_Avail}_i, \quad \forall i \quad (7)$$

where  $LT_i$  = the time required per aircraft launched in cycle  $i$ , and  
 $RT_i$  = time required to recover and rearm each aircraft recovered in cycle  $i$ .

The next constraint enforces the aircraft carrier's primary objective of completing its transit to the assigned location at the right time. We express this constraint in terms of the *shortfall*, which is the distance between the carrier's actual and intended locations at the specified transit completion time. The *shortfall* can be positive or negative: positive when the carrier arrives late at the assigned location, negative when it is early. (Being too early is less likely to happen and easier to rectify in practice than being too late.)

**Do not exceed the maximum allowed deviations from the assigned location.**

$$\begin{aligned} \text{Shortfall}(x) &\leq \text{Max\_pos\_deviation} \\ -\text{Shortfall}(x) &\leq \text{Max\_neg\_deviation} \end{aligned} \quad (8)$$

The notation  $\text{shortfall}(x)$  is meant to convey that the shortfall distance is actually a complicated nonlinear function of all the launch decision variables. This is because the final position of the carrier depends crucially on the selection and scheduling of all launches and recoveries. In the next section we describe a linear approximation of this function that renders the model solvable as a linear integer program. The approximation relies on the use of the control program in a pre-optimization analysis phase.

We have also implemented extensions of Constraint (8) which, at the commander's discretion, limit the carrier's deviation from PIM at the end of all cycles, not just the last cycle. Staying close to PIM throughout the transit may be important for coordinating with other vessels, particularly those with which communication is limited, such as submarines. Another reason why the commander may wish to stay close to PIM is navigational restrictions due to territorial boundaries or obstacles such as islands, shoals, and oil rigs.

#### **4. Linear Approximation of the PIM Proximity Constraint**

The most challenging aspect of the formulation of this problem as an optimization model is the development of an approximation of the PIM proximity constraint (8). The approximation needs to be sufficiently realistic and, at the same time, computationally tractable. For explanatory purposes, we describe its development in two stages. First, we treat in detail the carrier's movements during a single cycle of flight operations, and then we combine multiple applications of the single-cycle analysis for the general problem.

#### 4.1 Detailed Analysis of a Single Flight Operations Cycle

The movement of the carrier during cycle  $i$  is partitioned into four components, as depicted in Figure 2 and described in Table 1. Point  $A_i$  is the starting point, point  $F_i$  is the desired finish point, and Point  $E_i$  is the actual finish point at the end of the cycle. Ray  $A_iF_i$  represents the PIM. (In practice, PIMs are actually piece-wise linear; our program treats each linear piece in the manner described here.)

**TABLE 1: Components of Aircraft Carrier's Motion in Cycle  $i$**

<i>Component</i>	<i>Purpose</i>	<i>Relevant Parameters and Variables</i>	<i>Projection on PIM</i>
Arc $A_iB_i$	Turn into wind for flight operations.	$\theta_i$ = turn angle (depends on wind direction) $r_{i1}$ = turn radius for turn into wind $t_{i1}$ = time required for turn into wind	$d_{i1}$
Line $B_iC_i$	Conduct flight operations.	$x_{ijk}$ = flight operations variables $t_{i2}$ = time required for flight operations $LT_i$ = time per launch $RT_i$ = time per recovery $V_{i,launch}$ = velocity during flight operations (depends on wind velocity)	$d_{i2}$
Arc $C_iD_i$	Turn back towards PIM intercept.	$\mu_i$ = intercept angle $\phi_i$ = turn angle = $\mu_i + \theta_i$ $r_{i3}$ = turn radius for turn back $t_{i3}$ = time required for turn back	$d_{i3}$
Line $D_iE_i$	Sprint to PIM intercept.	$t_{i4}$ = sprint time (time remaining in cycle) $V_{i,sprint}$ = velocity during sprint	$d_{i4}$

The length of line  $E_iF_i$  represents the shortfall at the end of the cycle, which can be computed by the following system of equations. The key idea is to derive the projection of each component's motion on the PIM.

$$Shortfall = \frac{PIM\_distance - \sum_{h=1}^4 d_{ih}}{\cos \mu_i} \quad (9)$$

where

$$d_{i1} = r_{i1} \sin \theta_i \quad (10)$$

$$d_{i2} = t_{i2} V_{i,launch} \cos \theta_i \quad (11)$$

$$t_{i2} = LT_i \sum_{jk} x_{ijk} + RT_i \sum_j (x_{i-1,j,SC} + x_{i-2,j,DC} + x_{i-3,j,TC}) \quad (12)$$

$$d_{i3} = r_{i3} (\sin \theta_i + \sin \mu_i) \quad (13)$$

$$d_{i4} = t_{i4} V_{i,sprint} \cos \mu_i \quad (14)$$

$$t_{i4} = LC_i - t_{i1} - t_{i2} - t_{i3} \quad (15)$$

By repeated substitution, this system can be reduced to a single equation that expresses the shortfall as a function of the decision variables  $x_{ijk}$ . That equation is nonlinear, and hence unusable in our linear integer programming model, unless  $\theta_i$ ,  $\mu_i$ ,  $r_{i1}$ ,  $t_{i1}$ ,  $r_{i3}$ ,  $t_{i3}$ , and  $V_{i,launch}$  are known in advance. With the exception of the intercept angle,  $\mu_i$ , all of these terms depend directly on the wind and sea state, and they can be decided independently of flight operations decisions. Therefore, our model treats  $\theta_i$ ,  $r_{i1}$ ,  $t_{i1}$ ,  $r_{i3}$ ,  $t_{i3}$ , and  $V_{i,launch}$  as input parameters, relying on the best efforts of the carrier's meteorologists. (We also rely on the fact that the user of the model is the flight operations officer, whose job prerequisites include sufficient seamanship to be able to specify these parameters once the wind velocity and sea state are established.)

The nonlinearity induced by the PIM-intercept angle,  $\mu_i$ , is more difficult to cope with, because, in reality,  $\mu_i$  cannot be decided independently of flight operations decisions. This is because the commander's choice of sprint direction depends on the ship's position at the completion of flight operations and on the amount of time remaining in the cycle. These factors, in turn, depend on the number of launches and recoveries scheduled in the cycle. The commander may also base the choice of sprint direction on tactical considerations, e.g., avoiding obstacles, meeting with submarines, or reaching a desired location for the start of the next cycle.

From the point of view of mathematical interrelatedness, it may seem preferable to include  $\mu_i$ 's as decision variables in the model. However, the resulting nonlinear integer program would be beyond the ability of available solvers. Furthermore, this extension of the model might possibly interfere with the commander's tactical considerations. The approach we have taken is to run the control program portion of the decision support system with fixed values of the decision variables, and thereby obtain values of the intercept angles based on the user's informed judgement and the trial-and-error learning facilitated by the program's ability to rapidly evaluate alternatives. This is, of course, a heuristic approach dictated by the limits of technology in real-world applications. Nevertheless, we are optimistic that very little optimality is lost, since the choice of sprint direction depends on the *number* of flight operations in the cycle, rather than the precise, detailed mix of aircraft scheduled for launch and recovery.

## 4.2 Multi-Cycle Analysis

We now treat the general case of a multi-cycle transit, which is represented in Figure 3. This transit can be viewed as several interconnected repetitions of the four components of a single cycle, where the finish point  $E_i$  of one cycle is also the start point  $A_{i+1}$  of the next. At the end of the final cycle, the carrier's position is  $E_1$ , and the distance from the intended location can be computed by a simple generalization of the previous analysis.

$$\text{Shortfall} = \frac{\text{PIM\_distance} - \sum_{i=1}^I \sum_{h=1}^4 d_{ih}}{\cos \mu_I} \quad (16)$$

where

$$d_{i1} = r_{i1}(\sin \theta_i + \sin \mu_{i-1}) \quad (17)$$

and  $\mu_0=0$ , while  $d_{i2}$ ,  $d_{i3}$  and  $d_{i4}$  are computed as before. The extra term in equation (17), as compared to equation (10), accounts for the carrier's turn back to the PIM bearing after the previous cycle's sprint. We obtain a linear approximation of the shortfall to insert in constraint (8) by repeated substitution of this system and the use of the weather-dependent inputs described above.

## 5. Sample Scenario

We consider the following hypothetical scenario for illustrating the optimization model. A Carrier Battle Group is about to depart Pearl Harbor, Hawaii, en route to the Bering Sea. There will be four flight operations cycles of 105 minutes each. The first cycle commences at 1200 on 30 July. The transit is scheduled to finish at 0100 on 31 July, at which time the aircraft carrier must be within 20 nautical miles (NM) of a specified final point. The area en route and surrounding the final point will be sanitized by flights from the carrier to determine whether unfriendly submarines are shadowing the battle group.

The PIM of concern is a straight, northerly line whose endpoints are as follows:

<u>Time</u>	<u>Date</u>	<u>Lat.</u>	<u>Long.</u>	<u>Remarks</u>
1200	30 July	25-40N	159-00W	flight operations begin
0100	31 July	26-45N	159-00W	sanitation rendezvous

The minimum defensive posture, weather data and carrier speed requirements are provided in the sample worksheet in the Appendix. Data for the sample scenario in the notation of the optimization model are given in Table 2. The CVBG Commander is not concerned with position at the end of each cycle, only with meeting the 20 NM requirement at the end of the transit. Relaxation of any constraints is not authorized. The maximum number of discretionary sorties to launch per cycle of any aircraft type is 4. The maximum number of launch and recovery operations per cycle is 30, of which no more than 25 can be tactical launches.

**TABLE 2: Parameters for Sample Scenario**

<i>Parameter</i>	<i>Units</i>	<i>Cycle 1</i>	<i>Cycle 2</i>	<i>Cycle 3</i>	<i>Cycle 4</i>	<i>Cycle 5</i>
$LC_i$	minutes	105	105	105	105	360
$LT_i$	minutes	1.042	1.042	1.042	1.042	1.042
$RT_i$	minutes	4.456	4.456	4.456	4.456	4.456
$\theta_i$	radians	0.436	0.524	0.436	0.611	0.436
$r_{11}$	nautical miles	0.494	0.494	0.494	0.494	0.494
$t_{11}$	minutes	1.191	1.447	1.263	1.777	1.401
$V_{i, launch}$	knots	15	15	10	15	15
$\mu_i$	radians	0.007	0.026	0.040	0.077	0.109
$r_{13}$	nautical miles	0.494	0.494	0.494	0.494	0.494
$V_{i, sprint}$	knots	20	20	20	20	20
$t_{13}$	minutes	1.209	1.502	1.300	10877	1.488

Our optimization model is generated with GAMS and solved with Sunset Software's (1993) XA optimizer on a 486/66 personal computer. Through extensive use of variable and constraint reduction techniques within GAMS, the integer programming model has always been small enough to solve rapidly. In this instance, the problem was formulated with 178 general integer variables, 119 continuous variables and 186 constraints. It was solved by XA in 6 seconds.

Manually inputting the raw data into the control program took the user about 15 minutes. Once the data was entered, the combined computing time of the control program, GAMS and XA required for creating the model data, generating and solving the integer program, and reporting the optimal solution was under 15 seconds.

The quality of the optimized schedule was evaluated via comparison to a manual solution that assigns a single-cycle sortie for each required mission, as a CVBG flight operations officer might do in practice. The results are shown in Table 3. The table shows the number of launches and recoveries in each of the four flight operations cycles under both manual and optimized scheduling.

The manual and optimized solutions both satisfy the defensive posture requirements. Thanks to multi-cycle sorties, however, the optimized schedule provides the same airborne coverage with 36% fewer launches. The most important result is that the manual solution, in contrast to the optimizer, fails to reach the rendezvous area on time. At the end of the sprint,

the manual solution has the carrier 28 nautical miles from the specified final point, whereas the optimizer puts it within the 20 NM requirement.

Both solutions show, perhaps surprisingly to the reader, a recovery in the first cycle and a launch after the last cycle. These operations correspond to non-discretionary tanker flights, which do not have to be considered explicitly by the optimization model but are accounted for and reported in the control program. The CVBG keeps a tanker airborne during launches so that it can augment fuel levels of aircraft that have long missions to fly but need to take off from the carrier deck with low weight. The tanker's role during recovery periods is to be available for emergency mid-air refueling.

In this simple scenario with a straight-line PIM, the model yielded more protection for the carrier and earlier arrival at the final position, as compared with the likely outcome of traditional manual scheduling methods. For more complex scenarios, the potential benefits of the optimization model are even greater.

**TABLE 3: Comparison of Manual and Optimized Solutions**

<u>Manual Solution</u>							
<u>Cycle</u>	<u>Start Time</u>	<u>Stop Time</u>	<u>Num Launch</u>	<u>Num Rec</u>	<u>PIM Position</u>	<u>Planned Position</u>	<u>Deviation From PIM</u>
1	1200	1345	15	1	23-12N 158-57W	23-12N 159-00W	2
2	1345	1530	15	14	23-38N 158-48W	23-45N 159-00W	13
3	1530	1715	14	14	24-00N 158-44W	24-18N 159-00W	23
4	1715	1900	14	15	24-25N 158-34W	24-51N 159-00W	35
Final	1900	1931	1	15	24-39N 158-28W	25-11N 159-00W	43
After Sprint					26-17N 158-53W	26-45N 159-00W	28
Totals: 59 sorties, 101.5 airborne hours							

<u>Using Optimization Model</u>							
<u>Cycle</u>	<u>Start Time</u>	<u>Stop Time</u>	<u>Num Launch</u>	<u>Num Rec</u>	<u>PIM Position</u>	<u>Planned Position</u>	<u>Deviation From PIM</u>
1	1200	1345	15	1	23-12N 158-57W	23-12N 159-00W	2
2	1345	1530	11	10	23-40N 158-51W	23-45N 159-00W	9
3	1530	1715	6	6	24-09N 158-50W	24-18N 159-00W	13
4	1715	1900	10	11	24-37N 158-43W	24-51N 159-00W	21
Final	1900	2009	1	15	24-50N 158-37W	25-11N 159-00W	29
After Sprint					26-29N 158-56W	26-45N 159-00W	16
Totals: 38 sorties, 101.5 airborne hours							



## 6. Conclusion

The optimization model developed here gives rapid, face-valid, high-quality solutions to an important Navy problem. Like many problems in logistics, the problem involves the balancing of conflicting needs -- in this case, an aircraft carrier's need for self-protection vs. its requirement for on-time completion of an assigned transit.

It is important to point out that the model's results are best regarded as guidelines and bounds on the user's options, rather than as incontrovertible orders. The decision support system is designed with this sort of usage in mind.

## References

Brooke, A., D. Kendrick and A. Meeraus, *GAMS: A User's Guide, Release 2.25*, South San Francisco, CA: The Scientific Press, 1992.

Sunset Software Technology, *XA Professional Linear and General Integer Programming System, Version 2.2*, San Marino, California, 1993.

## APPENDIX

### CARRIER OPTIMIZATION LAUNCH MODEL WORKSHEET

A. DATE and TIME flight operations begin: Date 93211 Time 1200\_  
 LATITUDE/LONGITUDE at begin of flt ops: Lat 22-40N  
 Long 159-00W

B. DATE and TIME of next flight operations  
 period or rendezvous point: Date 93212 Time 0100\_  
 LATITUDE/LONGITUDE at end/rendezvous: Lat 26-45N  
 Long 159-00W

C. MAXIMUM SPRINT VELOCITY WHEN NOT IN FLT OPS: 20

D. DATE and TIME first flt ops period ends:

Date 93211 Time 1900\_  
 Number of Cycles: 4 All same length  
 If same length, length is: 105

E. MAXIMUM VELOCITY BETWEEN CYCLES: 20

F. MAXIMUM VELOCITY ON TURNS: 15

G. Distances allowed to deviate from PIM at rendezvous point:

BEHIND 20 AHEAD 20

#### H. WEATHER Data

Cycle	Time	W/V	SS	Vis	Ceil
1	1200	025/10	2	10	5k
2	1345	030/10	2	10	4.5k
3	1530	025/15	3	7	3k
4	1715	035/10	2	5	4k
5	1900	025/10	1	5	4k

#### I. SUNRISE / SUNSET Data

Date	Sunrise	Sunset
93211	0500	2030
93212	0501	2029

#### J. AIRCRAFT AVAILABLE

F14	F18	A6	EA6	E2	S3	ES3	TKR	COD	SH3
20	18	10	4	4	4	2	4	0	4

#### K. MINIMUM HOURS REQUIRED BY A/C TYPE

F14	F18	A6	EA6	E2	S3	ES3	TKR	COD	SH3
0	0	0	0	0	0	0	0	0	0

#### L. MAXIMUM CYCLE LENGTH BY A/C TYPE

F14	F18	A6	EA6	E2	S3	ES3	TKR	COD	SH3
2	2	2	2	2	2	2	1	0	0

**M. FIXED TACTICAL SORTIES BY A/C TYPE AND CYCLE LENGTH**  
(single cycle / double cycle / triple cycle)

	F14	F18	A6	EA6	E2	S3	ES3	TKR	COD	SH3
1	/	/	/	/	/1/	/	/	/	/	/
2	/	/	/2/	/	/	/	/	/	/	/
3	/	/	/1/	/	/	/	/	/	/	/
4	/	/	/	/	/	/	/	/	/	/
5	/	/	/	/	/	/	/	/	/	/

**N. AIRCRAFT REQUIRED AIRBORNE BY TYPE AND CYCLE**

	F14	F18	A6	EA6	E2	S3	ES3	TKR	COD	SH3
1	2	4	2	1	1	2	1	0	0	2
2	2	4	2	1	1	1	1	0	0	2
3	2	4	2	1	1	1	1	0	0	2
4	2	4	2	1	1	2	1	0	0	2
5										

**O. FIXED LOGICAL SORTIES BY A/C TYPE**  
(non-originating / non-returning / yo-yo)

	F14	F18	A6	EA6	E2	S3	ES3	TKR	COD	SH3
1	/	/	/	/	/	/	/	/1	/	/
2	/	/	/	/	/	/	/	/1	/	/
3	/	/	/	/	/	/	/	/1	/	/
4	/	/	/	/	/	/	/	/1	/	/
5	/	/	/	/	/	/	/	/1	/	/

P. MAXIMUM DISCRETIONARY SORTIES BY A/C TYPE (default 5): 5

Q. MAXIMUM TACTICAL SORTIES ALLOWED PER CYCLE (default 25): 25

R. MAXIMUM FLIGHT OPERATIONS ALLOWED PER CYCLE: 30

S. DISTANCE ALLOWED BEHIND AT END OF EACH CYCLE: N/A

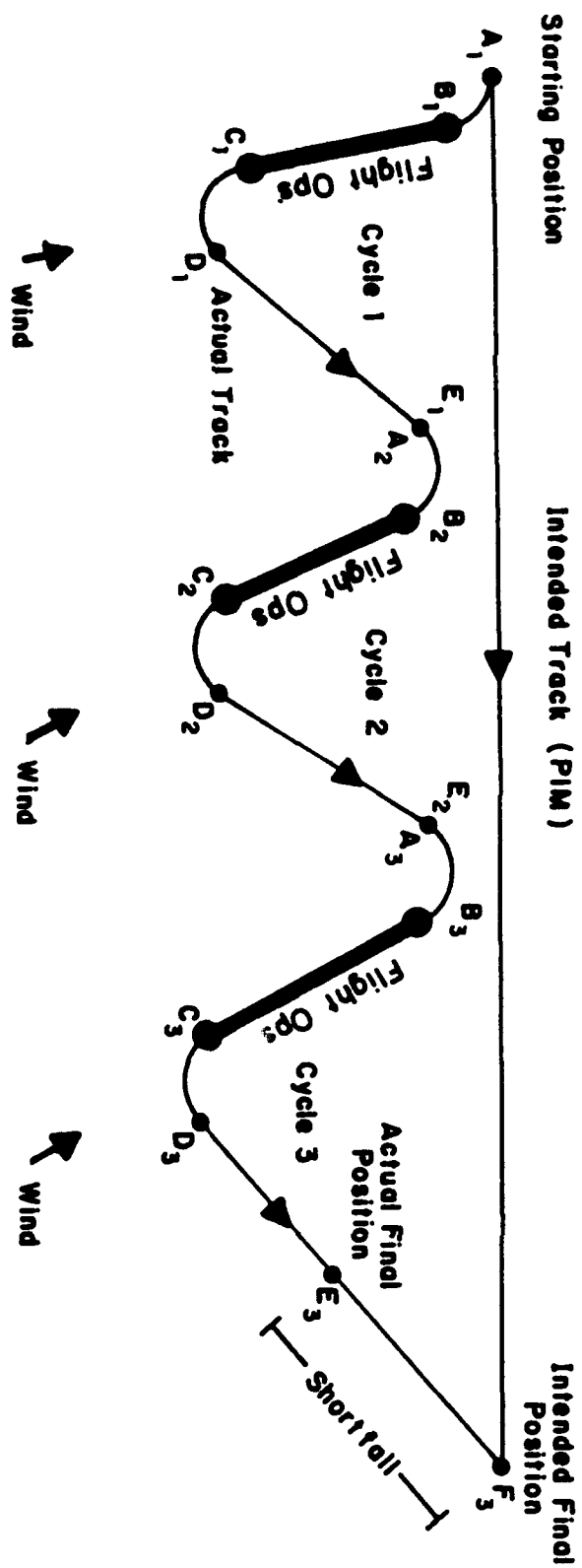


Figure 1: Conflicting objectives: flight operations are needed for the aircraft carrier's self-defense but they cause deviations from the position of intended motion (PIM).



**Figure 2: Components of the aircraft carrier's motion in a single cycle.**

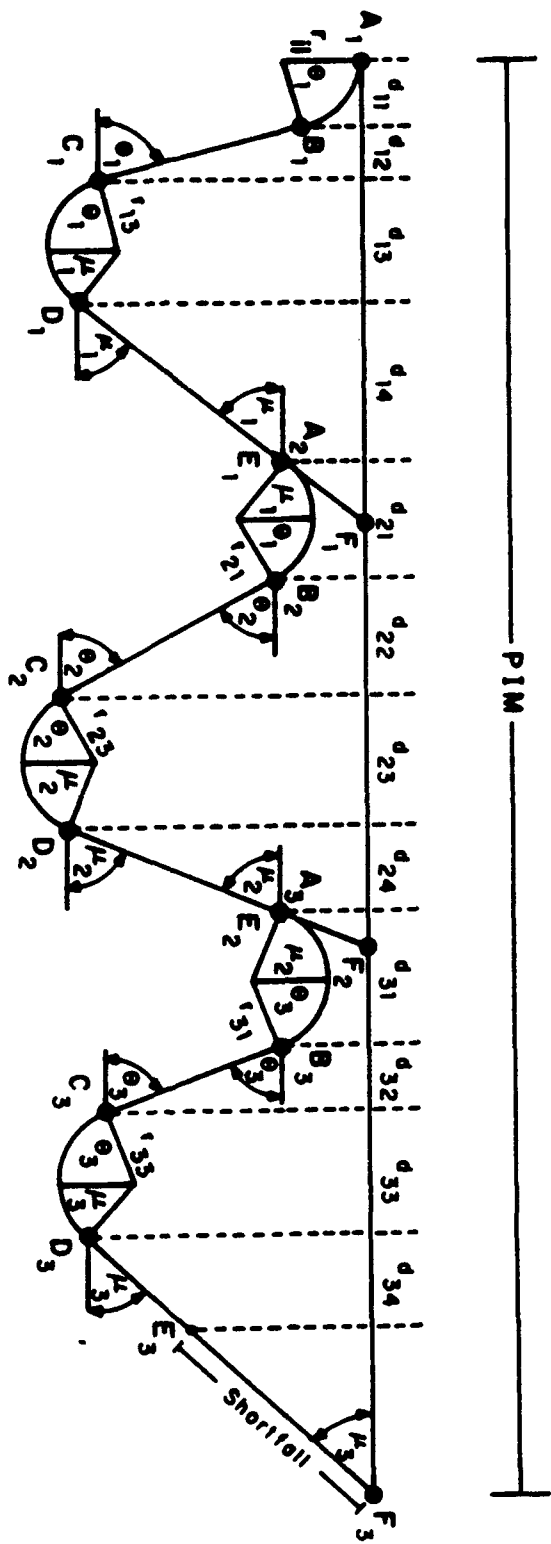


Figure 3: Components of a multi-cycle transit.

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